Mid-frequency vibro-acoustic modelling at INSA Lyon

A focus on SmEdA model

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Where we are?

PARIS

ALPS

LYON

FRENCH RIVIERA
INSA Lyon: An engineering school

5,400 students

1,300 graduates every year, with 900 engineers in 12 specialist fields

659 researcher-teachers and researchers

21 laboratories
Staff:
- 16 professors and assistant-professors
- 1 research engineer
- 4 post-doctoral researchers
- 22 Ph.D. students

Research activities:
- Vibro-acoustics
- Source identification
- Sound and vibration perception
- Monitoring NDT - SHM

Evaluation from the national agency: A+
Extension of SmEdA to non-resonant transmission

« Vibro-acoustics »

- Energetical methods
- PTF
- Stiffened structures
- Fluid-structure interaction in heavy fluids
- Turbulent boundary layer
- Micro electro. mech. systems

[Images of various methods and applications related to vibro-acoustics]
« Source identification – inverse problems »

- Identification of acoustical or vibratory sources
- Model identification
- Source characterization
« Sound and vibration perception »

- Proposal of accurate metrics
- Influence of mechanical parameters on sound perception
- Whole-body vibration, interaction between sound and vibration
Material and structures characterization

Monitoring from vibration or rotational speed measurements

Defect detection and localisation (ultrasound or X-rays).
Some on-going projects

**Mid-frequency problems**: Acoustic optimization of a low-weight truck cabin (*CLIC project, National and European grants*)

**Source identification**: Vibro-acoustic beamforming for leak detection on SFR steam generator (*CEA – AREVA*)

**Sound perception**: Electric vehicle warning sounds (*European projet EVADER*)

**Micro electro mechanical systems**: digital loudspeaker (with CEA-Leti), application for hearing aids (*ANR Madnems*)

**Non destructive testing**: Data fusion from X-rays and ultrasound images (*European projet FFRESHeX*)
CeLyA: Cluster of laboratories working on acoustics

- 70 people, members from 8 teams in Lyon (LVA, LMFA, LTDS, DGCB, CREATIS, LABTAU, CRNL, IFSTTAR).
- Research fields:
  - Vibro-acoustics, aero-acoustics, non-linear acoustics, ultra-sounds (NDT and medical applications), psycho-acoustics, noise effects…

http://celya.universite-lyon.fr
Vibro-acoustic modelling in the mid-frequency domain
Vibro-acoustic modelling of complex mechanical structures

Response

Frequency

- BF
- MF
- HF

FEM / BEM
IEM / ...

PTF
Patch Transfer Function

SmEdA
Statistical modal Energy distribution Analysis
Statistical modal Energy distribution Analysis

I – Dual Modal Formulation
II – Fundamentals of SmEdA
III – Interests of SmEdA
IV – Extension to non-resonant transmission
V – Methodology for including dissipative threatsments
VI – Modelling of the vibration transmission through industrial structures
I. Dual Modal Formulation (DMF) for coupled subsystems

Two coupled SEA subsystems

Cavity-Structure: [Fahy69,70]
Two coupled rods: [Karnopp64]
General case: ?

Modal coupling schema suggested by SEA

Subsystem 1

DMF

Subsystem 2

Uncoupled – blocked subsystem

$W=0$

$\sigma=0$


$(\omega_q, \sigma_{ij}^q), \forall q \in [1, N_2]$

$(\omega_p, W_i^p), \forall p \in [1, N_1]$
I. Dual Modal Formulation (DMF) for coupled subsystems

An example

Two coupled beams

Dash, DMF with only the **resonant** modes

Full, Reference

Uncoupled subsystem modes and intermodal works

\[ W_{pq}^{12} = \tilde{\Theta}_z^{1F}(L_1) \tilde{M}_f^{2q}(L_1) \]

Energy spectrum for the non excited beam.

⇒ a mechanical impedance mismatch is need
II. Fundamentals of Statistical modal Energy distribution Analysis

SmEdA is based on:

- the uncoupled-subsystem modes (natural frequency, mode shape at coupling surfaces)
- the same assumptions as SEA except for the modal energy equipartition
- the description of the energy sharing between modes rather than between subsystems

- Power balance mode $p$:

\[
\Pi_{inj}^p = \Pi_{diss}^p + \sum_{q'=1}^{N_2} \Pi_{pq'}^{12}
\]

- Modal injected power
- Modal dissipated power
- Powers exchanged with modes of other coupled subsystems
II. Fundamentals of Statistical modal Energy distribution Analysis

- Energy sharing between mode \( p \) and mode \( q \)

\[
P_{12} = \beta \left( E_1 - E_2 \right)
\]

Modal Coupling Loss Factors (MCLFs):

Intermodal work

\[
\beta_{pq}^{12} = \frac{\left( W_{pq}^{12} \right)^2}{M_p^1 \left( \omega_q^2 \right)^2 M_q^2} \left\{ \frac{\Delta_p^1 \left( \omega_q^2 \right)^2 + \Delta_q^2 \left( \omega_p^1 \right)^2}{\left[ \left( \omega_p^1 \right)^2 - \left( \omega_q^2 \right)^2 \right]^2 + \left( \Delta_p^1 + \Delta_q^2 \right) \left[ \Delta_p^1 \left( \omega_q^2 \right)^2 + \Delta_q^2 \left( \omega_p^1 \right)^2 \right]} \right\}.
\]

Modal frequency

Modal damping
II. Fundamentals of Statistical modal Energy distribution Analysis

• Modal energy equations of motions (for the two coupled subsystems)

\[
\Pi_{\text{inj}}^{1p} = \left( \omega_p^1 \eta_p^1 + \sum_{q'=1}^{N_2} \beta_{pq}^{12} \right) E_p^1 - \sum_{q'=1}^{N_2} \beta_{pq'}^{12} E_q^1, \quad \forall p \in \{1, \ldots, N_1\},
\]

\[
\Pi_{\text{inj}}^{2q} = - \sum_{p'=1}^{N_1} \beta_{p'q}^{12} E_{p'}^1 + \left( \omega_q^2 \eta_q^2 + \sum_{p'=1}^{N_1} \beta_{p'q}^{12} \right) E_q^2, \quad \forall q \in \{1, \ldots, N_2\}.
\]

\[\Rightarrow N_1 + N_2 \text{ equations, } N_1 + N_2 \text{ unknowns} \Rightarrow \text{SmEdA}\]

• Modal energy equipartition assumption (i.e. \( E_p^1 = \frac{E}{N_1} \) \( \forall p \in \{1, N_1\} \), etc)

\[\Rightarrow \text{SEA equations with the Coupling Loss Factors (CLFs):}\]

\[\eta_{12} = \frac{\sum_{\alpha=1}^{N_1} \sum_{\sigma=1}^{N_2} \beta_{\alpha\sigma}^{12}}{N_1 \omega_c}\]

depending only on the modal information of each uncoupled subsystem
III. Interests of Statistical modal Energy distribution Analysis

(1) Subsystems with low modal overlap
- Low damping
- Low modal density
- Mid-frequency domain

Example on a case studied in the literature: Two coupled beams with varying damping
FF. YAP, J. WOODHOUSE - Investigation of damping effects on statistical energy analysis of coupled structures, JSV, 197 (1996)

III. Interests of Statistical modal Energy distribution Analysis

(2) Heterogeneous subsystems

Example: Four coupled beams

Substructuring in 3 subsystems (by a silly student!)

Energy ratio $E_4/E_1$
- FEM: -38.3 dB
- SEA: -24.0 dB
- SmEdA: -36.3 dB

Example of two modes shapes of subsystem 2

Modal energy distribution for subsystem 2 (1000 Hz octave band)

(3) Spatially localised excitations

(4) Hybrid model SEA/SmEdA

III. Interests of Statistical modal Energy distribution Analysis

(5) Estimation of CLFs for complex subsystems
(SmEdA with equipartition assumption → SEA-like)

« Classical-historical » approach
S-S 1 ~ plate
S-S 2 ~ shell
Line coupling

η₁₂ obtained from the travelling wave approach

SmEdA approach
Calculation of the normal modes of each uncoupled-subsystem with FEM

η₁₂ deduced from the analytical expression depending on the mode information (i.e. frequency, shape, loss factor)
Three. Interests of Statistical modal Energy distribution Analysis

Comparison with the travelling wave approach on basic cases

Two coupled steel panels at right angle

Subsystem 1
10 mm

Subsystem 2
30 mm

Subsystem 3
40 mm

Subsystem 2
20 mm

Cylindrical shell coupled to two end panels

Frequency (Hz)

CLFs

Frequency (Hz)

CLFs
III. Interests of Statistical modal Energy distribution Analysis

Industrial applications developed in the past

Vibration transmission through car firewall (RENAULT - 2001)

Vibration transmission through car floor (FIAT - 2002)

Sound radiation from car structure (2009)

N. Totaro, C. Dodard, J.L. Guyader, SEA Coupling Loss Factors of Complex Vibro-Acoustic Systems, JVA, ASME, 2009
IV. Extension of SmEdA to non-resonant transmission

Context: Evaluation and analysis of the noise transmission through truck cab structures

Question 1: What is the TL of the floor when the emitting cavity is the engine compartment and the receiving cavity is the truck cabin?

- Influence of the sizes and shapes of the cavities on the noise transmission
- Interest for a predictive method to estimate the TL of complex structures taking the behavior of small cavities and the structure geometry into account.
IV. Extension of SmEdA to non-resonant transmission

Estimation of the TL using Statistical Energy Analysis (SEA) model (for high frequency).

\[ \text{For } f > f_c \text{ (i.e. resonant transmission), the classical SEA model can be applied.} \]

\[ \text{For } f < f_c, \text{ Crocker and Price, JSV 9 (1969) proposed a modified SEA model to} \]
\[ \text{take the non resonant transmission into account.} \]

Question 2: How can we estimated this CLF for a structure with a complex geometry?

Direct coupling of the 2 cavities

\[ \text{Mass law transmission} \]

\[ \text{CLF estimated for infinite plate (Crocker), double panel (Cray).} \]
IV. Extension of SmEdA to non-resonant transmission

Transmission Loss problem for complex cavity

Modal interaction
(Black, resonant transmission, red, non resonant transmission)
IV. Extension of SmEdA to non-resonant transmission

The Dual Modal Formulation (DMF) allows us to write the matrix system:

\[
\begin{bmatrix}
Z_{11} & -j\omega W_{12} & 0 & \Gamma_1 \\
+j\omega W_{12}^* & Z_{22} & j\omega W_{23}^* & \Gamma_2 \\
0 & -j\omega W_{23} & Z_{33} & \Gamma_3 \\
\end{bmatrix}
\begin{bmatrix}
Q_1 \\
0 \\
0 \\
\end{bmatrix}
\]

Considering two sets of modes for the structure: the resonant (R) and the non-resonant (NR) gives:

\[
\begin{bmatrix}
Z_{11} & -j\omega W_{12}^{NR} & -j\omega W_{12}^R & 0 & \Gamma_1^{NR} & \Gamma_1^R & \Gamma_1 \\
+j\omega W_{12}^{NR*} & Z_{22}^{NR} & 0 & -j\omega W_{23}^{NR} & 0 & 0 & 0 \\
+j\omega W_{12}^{R*} & 0 & Z_{22}^R & +j\omega W_{23}^{R*} & 0 & 0 & 0 \\
0 & -j\omega W_{23}^{NR} & -j\omega W_{23}^R & Z_{33} & \Gamma_3^{NR} & \Gamma_3^R & \Gamma_3 \\
\end{bmatrix}
\begin{bmatrix}
Q_1 \\
0 \\
0 \\
0 \\
0 \\
\end{bmatrix}
\]

Achieving a condensation on the NR modes and assuming mass controlled behaviour for these modes, we obtain:

\[
\begin{bmatrix}
Z_{11} & -j\omega W_{12}^R & -W_{12}^{NR} W_{23}^{NR*} & \Gamma_1 \\
+j\omega W_{12}^{R*} & Z_{22} & +j\omega W_{23}^{R*} & \Gamma_2^R \\
-W_{23}^{NR} W_{12}^{NR*} & -j\omega W_{23}^R & Z_{33} & \Gamma_3 \\
\end{bmatrix}
\begin{bmatrix}
Q_1 \\
0 \\
0 \\
\end{bmatrix}
\]

IV. Extension of SmEdA to non-resonant transmission

Direct modal coupling factor between the two cavities:

\[
\beta_{pr} = \left( \sum_{q \in Q_{NR}} W_{pq} W_{rq} \right)^2 \left\{ \frac{(\omega_p \eta_p + \omega_r \eta_r)}{\left[ \left( \omega_p \right)^2 - \left( \omega_r \right)^2 \right]^2 + (\omega_p \eta_p + \omega_r \eta_r) \left[ \omega_p \eta_p \left( \omega_p \right)^2 + \omega_r \eta_r \left( \omega_p \right)^2 \right]} \right\}.
\]

p : modes of cavity 1  
r : modes of cavity 2  
q : non-resonant modes of the plate

\( W_{pq} \) and \( W_{rq} \) are the intermodal works between modes of the cavities and non-resonant modes of the structure

SEA coupling loss factor between the 2 cavities:

\[
\eta_{C1-C2} = \sum_{p \in P} \sum_{r \in R} \beta_{pr}
\]

IV. Extension of SmEdA to non-resonant transmission

![Diagram of Cavity A and Cavity B with a monopole and a plate, labeled hp]

- SmEdA without NR modes
- SmEdA with NR modes
- Reference

![Graph showing energy ratio (dB) vs. frequency (Hz) with three traces]

- Frequency axis ranges from $10^2$ to $10^3$ Hz
- Energy ratio axis ranges from 0 to 60 dB
IV. Extension of SmEdA to non-resonant transmission

TL between two “Small” cavities

Rectangular thin plate (1mm of thickness)

0.6 m

0.7 m

Intermodal works (Third octave band 1000 Hz)
IV. Extension of SmEdA to non-resonant transmission

Modal transfer path analysis

Emitting cavity – panel (Resonant transmission)

Emitting cavity – Receiving cavity (NR transmission)

Modal coupling loss factors - 1000 Hz third octave

Modal energy distribution for the receiving cavity - 1000 Hz third octave
IV. Extension of SmEdA to non-resonant transmission

“Complex structure”

- Ribbed plate
  - Plate thickness: 1mm
  - Rib cross-section: square (5mm x 5mm)
  - Rib spacing: 50 mm

Finite element meshing of the ribbed plate
19481 Nodes, 19200 CQUAD4, 1800 CBEAM

Natural frequencies and mode shapes calculation until 10 kHz (MD NASTRAN)

- Analytical cavity modes
- SmEdA calculation by third octave band

Transmission Loss
IV. Extension of SmEdA to non-resonant transmission

Example of results:
- Plate thickness: 1mm
- Rib cross-section: square (5mm x 5mm)
- Rib spacing: 50 mm

![Graph showing Energy Noise Reduction $E_{C1}/E_{C2}$ (dB) vs Frequency (Hz)]
V. Methodology for including the effect of dissipative treatments in SmEdA

On-going project: CLIC (2012-2015 – 5.3 M€, 0.5 M€ for LVA)
- Acoustic optimization of a low-weight truck cabin
- Different partners: RENAULT TRUCK, ARCELOR MITTAL, ACOEM, FEMTO-ST, ALTRAN, A2MAC1

LVA works:
Develop a Mid-frequency modelling of acoustic radiation/transmission of the truck cab taking the effect of the dissipative treatments into account.

- Viscoelastic layers (Damping layer)
- Acoustic absorbing materials (trim, foarm)
V. Methodology for including the effect of dissipative treatments in SmEdA

Illustration of the methodology on the validation test case

Experimental set-up of the validation case
(Rectangular “clamped” plate radiating into a rectangular cavity with “rigid” wall)

V. Methodology for including the effect of dissipative treatments in SmEdA

Illustration of the methodology on the validation test case

**1st step:**
Characterisation of the equivalent dissipative material

**2nd step:**
Creation of FE model including the equivalent dissipative elements

V. Methodology for including the effect of dissipative treatments in SmEdA

Illustration of the methodology on the validation test case

3rd step:
Calculation of the normal modes and evaluation of the modal damping loss factors (from the imaginary part of the FE stiffness matrix)

\[ \eta_p = \frac{\text{Im}\left\{W_p \bar{K}W_p\right\}}{\text{Re}\left\{W_p \bar{K}W_p\right\}} \]

4th step:
Calculation of the modal coupling loss factors (MCLFs)

\[ \beta_{pq} = f(\omega_p, M_p, W_p, \eta_p, \omega_q, M_q, p_q, \eta_q) \]

5th step:
Calculation of the modal energies and the subsystem energy from SmEdA equations

\[ E_p, E_q \]

\[ E_{t1}, E_{t2} \]
V. Methodology for including the effect of dissipative treatments in mid-frequency SmEdA modelling

- Application/validation on the test truck cab on-going

VI. Modelling of the vibration transmission through industrial structures

Recent works achieved by ACOEM (acoustic consulting company):
- Automation of the numerical process through an in-house code (developed under ANSYS-APDL environment)

- Benchmark on a mock-up of a nuclear power plant structure

- Applications on industrial buildings

Conclusions
SmEdA overview
- SEA energy equipartition assumption relaxed
  ➔ Extension to low modal overlap subsystems
  ➔ Extension to non-homogeneous subsystems
- Method based on the uncoupled subsystems modes;
  ➔ Description of subsystems with complex
    geometry/mechanical properties (using FEM)
  ➔ Description of dissipative treatments
    (on-going research)
- Non-resonant transmission modelling
  ➔ Prediction of TL of complex structures in mid-frequency
- Hybrid SEA/SmEdA allow

Future research on SmEdA
- Uncertainty
- Path analysis / reduced model using the graph theory
Thank you for your attention