End-to-end statistical models for predicting maximum expected vibro-acoustic response levels under aero-acoustic loading

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In the space industry, it is necessary to “qualify” all equipment that will fly on a launch vehicle by vibration testing to maximum expected vibration levels. Piersol [1] has shown that there is considerable uncertainty in flight random vibration levels. An ensemble of measurements from multiple similar launch vehicles showed the vibration levels are approximately log normally distributed with a relative variance of approximately unity, as summarized in Figure 1 below. MIL-SPEC 1540 and NASA HBDBK 7005 recommend the use of (Log) Normal Tolerance Limits to define the maximum expected vibration test specification. This paper addresses the question of how to estimate maximum expected vibration levels when using a physics-based model to design the launch vehicle.

The simulation of linear vibration response under lift-off and ascent aeroacoustic loading, involves the surface pressure loading spectrum and its spatial correlation, and the modal dynamics of the loaded structure. Aeroacoustic loading is often temporally random and only partially space-correlated. Since random loading can only be described statistically, the structure vibration response can only be predicted statistically.

However, it is also common that the modal dynamics of the structure are quasi-random (eg. due to uncertainties in the as-built boundary conditions) particularly at higher frequencies. In this situation,
design for the maximum expected vibration or stress or strain response requires an “end-to-end” statistical model that correctly combines randomness in the loading with uncertainty in the structure dynamics. To this end, it is necessary to extend existing models to predict not only the statistical mean but also the statistical variance and probability density function for the end-to-end prediction process.

This paper will show that the total variance is a function of at least three statistically independent random variables – the fluctuating surface pressure spectrum \( \overline{p_s}^2 \), the modal power acceptance of the local structure modes \( j^2 \text{Re}\left[ M_r^\infty \right] \) and the modal damping \( Q_r \equiv 1/\eta_r \). Under reasonable assumptions, it will be shown that the total relative variance of vibration response \( r^2 \left[ v_s^2 \right] \) can be estimated as:

\[
r^2 \left[ v_s^2 \right] = r^2 \left[ \overline{p_s}^2 \right] + r^2 \left[ Q \right] + r^2 \left[ j^2 \text{Re}\left[ M_r^\infty \right] \right]
\]

(1.1)

Sample data ensembles – available in the open literature [2][3] – are used to show that this statistical model appears to be consistent with measured flight vibration variance, as shown in Figure 2 below.

![Graph showing comparison of component vibroacoustic model relative variances with total relative variance of flight vibration ensemble.](image)

**Figure 2** Comparison of component vibroacoustic model relative variances with total relative variance of flight vibration ensemble.

**References**

